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## Short Communication

# Note on “ $\mathcal{IP}$ -separation axioms in ideal bitopological ordered spaces $\Pi$ ”



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 $\mathcal{IP}$ -regular ordered space;  
 $\mathcal{IP}$ -completely normal ordered space

**Abstract** In this note, we show that Examples 3.1, 3.3, 3.4, 3.5, 3.8, 3.9, 3.10 and 3.11 in [1] are incorrect, by giving remarks and comments on these examples. Finally, reasonable reasons to improve some of the incorrect examples have been mentioned.

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## 1. Preliminaries

In this section, we recall some basic notions in ideal and ideal bitopological ordered spaces.

**Definition 1.1** [2]. A nonempty collection  $\mathcal{I}$  of subsets of a set  $X$  is called an ideal on  $X$ , if it satisfies the following assertions:

1.  $A \in \mathcal{I}$  and  $B \in \mathcal{I} \Rightarrow A \cup B \in \mathcal{I}$ , (finite additivity),
2.  $A \in \mathcal{I}$  and  $B \subseteq A \Rightarrow B \in \mathcal{I}$ , (heredity).

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**Definition 1.2** [3]. Let  $(X, R)$  be a poset and  $\mathcal{I}$  be an ideal on  $X$ . A set  $A \subseteq X$  is said to be:

1.  $\mathcal{I}$ -decreasing if  $Ra \cap A^c \in \mathcal{I} \forall a \in A$ , where  $Ra = \{b : bRa\}$  and  $A^c$  is the complement of  $A$ ,
2.  $\mathcal{I}$ -increasing if  $aR \cap A^c \in \mathcal{I} \forall a \in A$ , where  $aR = \{b : aRb\}$ .

**Definition 1.3** [4]. A space  $(X, \tau_1, \tau_2, R, \mathcal{I})$  is called an ideal bitopological ordered space if  $(X, \tau_1, \tau_2, R)$  is a bitopological ordered space and  $\mathcal{I}$  is an ideal on  $X$ .

**Definition 1.4** [4]. An ideal bitopological ordered space  $(X, \tau_1, \tau_2, R, \mathcal{I})$  is said to be:

1.  $\mathcal{I}$ -lower  $PT_1$  ( $ILPT_1$ , for short) ordered space if for every  $a, b \in X$  such that  $aRb$ , there exists an  $\mathcal{I}$ -increasing  $\tau_i$ -open set  $U$  such that  $a \in U$  and  $b \notin U$ ,  $i = 1$  or  $2$ .
2.  $\mathcal{I}$ -upper  $PT_1$  ( $IUPT_1$ , for short) ordered space if for every  $a, b \in X$  such that  $aRb$ , there exists an  $\mathcal{I}$ -decreasing  $\tau_i$ -open set  $V$  such that  $b \in V$  and  $a \notin V$ ,  $i = 1$  or  $2$ .
3.  $\mathcal{IPT}_1$ -ordered space if it is  $ILPT_1$  and  $IUPT_1$  ordered space.

**Definition 1.5 [1].** An ideal bitopological ordered space  $(X, \tau_1, \tau_2, R, \mathcal{I})$  is said to be:

1.  $\mathcal{I}$ -lower pairwise regular ( $\mathcal{ILPR}_2$ , for short) ordered space if for every  $\mathcal{I}$ -decreasing  $\tau_i$ -closed set  $F$  and for every  $a \notin F$ , there exist an  $\mathcal{I}$ -increasing  $\tau_i$ -open set  $U$  and an  $\mathcal{I}$ -decreasing  $\tau_j$ -open set  $V$  such that  $a \in U$ ,  $F - V \in \mathcal{I}$  and  $U \cap V \in \mathcal{I}$ .
2.  $\mathcal{I}$ -upper pairwise regular ( $\mathcal{IUPR}_2$ , for short) ordered space if for every  $\mathcal{I}$ -increasing  $\tau_i$ -closed set  $F$  and for every  $a \notin F$ , there exist an  $\mathcal{I}$ -decreasing  $\tau_i$ -open set  $U$  and an  $\mathcal{I}$ -increasing  $\tau_j$ -open set  $V$  such that  $a \in U$ ,  $F - V \in \mathcal{I}$  and  $U \cap V \in \mathcal{I}$ .
3.  $\mathcal{I}$ -pairwise regular ( $\mathcal{IPR}_2$ , for short) ordered space if it is  $\mathcal{ILPR}_2$  and  $\mathcal{IUPR}_2$ .

**Definition 1.6 [1].** An ideal bitopological ordered space  $(X, \tau_1, \tau_2, R, \mathcal{I})$  is called  $\mathcal{IPT}_3$ -ordered space if it is  $\mathcal{IPR}_2$  and  $\mathcal{IPT}_1$ -ordered space.

**Definition 1.7 [1].** Let  $(X, \tau_1, \tau_2, R, \mathcal{I})$  be an ideal bitopological ordered space and  $A, B \subseteq X$ . Then  $A$  and  $B$  are said to be  $\mathcal{IP}$ -separated sets if  $A \cap \tau_j - cl(B) \in \mathcal{I}$  and  $\tau_i - cl(A) \cap B \in \mathcal{I}$ .

**Definition 1.8 [1].** An ideal bitopological ordered space  $(X, \tau_1, \tau_2, R, \mathcal{I})$  is said to be  $\mathcal{IP}$ -completely normal ordered space ( $\mathcal{IPR}_4$ -ordered space, for short) if for any two  $\mathcal{IP}$ -separated subsets  $A$  and  $B$  of  $X$  such that  $A$  is  $\mathcal{I}$ -increasing set and  $B$  is  $\mathcal{I}$ -decreasing set there exist an  $\mathcal{I}$ -increasing  $\tau_i$ -open set  $U$  and  $\mathcal{I}$ -decreasing  $\tau_j$ -open set  $V$  such that  $A \subseteq U$ ,  $B \subseteq V$  and  $U \cap V \in \mathcal{I}$ .

## 2. Main results

Kandil et al. [Example 3.1, 1] claimed that  $(X, \tau_1, \tau_2, R, \mathcal{I})$  is  $\mathcal{ILPR}_2$ -ordered space, but this is erroneous by the following remark.

**Remark 2.1.** The family of all  $\mathcal{I}$ -decreasing  $\tau_1$ -closed sets is  $\{X, \{2\}, \{2, 3, 4\}\}$ , the collection of all  $\mathcal{I}$ -increasing  $\tau_1$ -open sets is  $\{X, \{4\}, \{1, 4\}, \{1, 3, 4\}\}$  and  $X$  is the only  $\mathcal{I}$ -decreasing  $\tau_2$ -open set. Hence  $F = \{2, 3, 4\}$  is  $\mathcal{I}$ -decreasing  $\tau_1$ -closed set not containing 1,  $U = X$  or  $\{1, 4\}$  or  $\{1, 3, 4\}$  is the only  $\mathcal{I}$ -increasing  $\tau_1$ -open set containing 1 and  $V = X$  is the only  $\mathcal{I}$ -decreasing  $\tau_2$ -open set such that  $F - V = \emptyset \in \mathcal{I}$  but  $U \cap V \notin \mathcal{I}$ .

Kandil et al. [Example 3.3, 1] claimed that  $(X, \tau_1, \tau_2, R, \mathcal{I})$  is  $\mathcal{IPR}_2$ -ordered space, but this is incorrect by the following remark.

**Remark 2.2.** The family of all  $\mathcal{I}$ -decreasing  $\tau_1$ -closed sets is  $\{X, \{3\}, \{4\}, \{3, 4\}\}$ , the collection of all  $\mathcal{I}$ -increasing  $\tau_1$ -open sets is  $\{X, \{1, 2, 3\}\}$  and  $\{X, \{2, 3\}\}$  is the family of all  $\mathcal{I}$ -decreasing  $\tau_2$ -open sets. Hence  $F = \{3, 4\}$  is  $\mathcal{I}$ -decreasing  $\tau_1$ -closed set not containing 1,  $U = X$  or  $\{1, 2, 3\}$  is the only  $\mathcal{I}$ -increasing  $\tau_1$ -open set containing 1 and  $V = X$  or  $\{2, 3\}$  is the only  $\mathcal{I}$ -decreasing  $\tau_2$ -open set such that  $F - V \in \mathcal{I}$  but  $U \cap V \notin \mathcal{I}$ . Hence  $(X, \tau_1, \tau_2, R, \mathcal{I})$  is not  $\mathcal{ILPR}_2$ -ordered space. As a result,  $(X, \tau_1, \tau_2, R, \mathcal{I})$  is not  $\mathcal{IPR}_2$ -ordered space.

Kandil et al. [Example 3.4, 1] asserted that  $(X, \tau_1, \tau_2, R, \mathcal{I})$  is  $\mathcal{IPT}_3$ -ordered space, but this is incorrect by the following remark.

**Remark 2.3.** The family of all  $\mathcal{I}$ -decreasing  $\tau_1$ -closed sets is  $\{X, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{2, 3, 4\}\}$ , the collection of all  $\mathcal{I}$ -increasing  $\tau_1$ -open sets is  $\{X, \{3\}, \{1, 3\},$

$\{3, 4\}, \{1, 2, 3\}, \{1, 3, 4\}\}$  and  $\{X, \{2, 3\}\}$  is the family of all  $\mathcal{I}$ -decreasing  $\tau_2$ -open sets. Hence  $F = \{2, 3, 4\}$  is  $\mathcal{I}$ -decreasing  $\tau_1$ -closed set not containing 1,  $U = X$  or  $\{1, 3\}$  or  $\{1, 2, 3\}$  or  $\{1, 3, 4\}$  is the only  $\mathcal{I}$ -increasing  $\tau_1$ -open set containing 1 and  $V = X$  or  $\{2, 3\}$  is the only  $\mathcal{I}$ -decreasing  $\tau_2$ -open set such that  $F - V \in \mathcal{I}$ , but  $U \cap V \notin \mathcal{I}$ . Hence  $(X, \tau_1, \tau_2, R, \mathcal{I})$  is not  $\mathcal{ILPR}_2$ -ordered space. As a result,  $(X, \tau_1, \tau_2, R, \mathcal{I})$  is not  $\mathcal{IPR}_2$ -ordered space. It follows that it is not  $\mathcal{IPT}_3$ -ordered space.

Kandil et al. [Example 3.5, 1] asserted that  $(X, \tau_1, \tau_2, R, \mathcal{I})$  is  $\mathcal{IPR}_2$ -ordered space, but this is incorrect by the following remark.

**Remark 2.4.** The family of all  $\mathcal{I}$ -decreasing  $\tau_1$ -closed sets is  $\{X, \{3\}, \{4\}, \{1, 4\}, \{3, 4\}, \{1, 3, 4\}\}$ , the collection of all  $\mathcal{I}$ -increasing  $\tau_1$ -open sets is  $\{X, \{2, 3\}, \{1, 2, 3\}\}$  and  $\{X, \{2, 3\}\}$  is the family of all  $\mathcal{I}$ -decreasing  $\tau_2$ -open sets. Hence  $F = \{1, 3, 4\}$  is  $\mathcal{I}$ -decreasing  $\tau_1$ -closed set not containing 2,  $U = X$  or  $\{2, 3\}$  or  $\{1, 2, 3\}$  is the only  $\mathcal{I}$ -increasing  $\tau_1$ -open set containing 2 and  $V = X$  or  $\{2, 3\}$  are the only  $\mathcal{I}$ -decreasing  $\tau_2$ -open set such that  $F - V \in \mathcal{I}$ , but  $U \cap V \notin \mathcal{I}$ . Hence  $(X, \tau_1, \tau_2, R, \mathcal{I})$  is not  $\mathcal{ILPR}_2$ -ordered space. Thus  $(X, \tau_1, \tau_2, R, \mathcal{I})$  is not  $\mathcal{IPR}_2$ -ordered space.

**Note 2.5.** It may be noted that [Example 3.5, 1] is not  $\mathcal{PR}_2$ .

The following remark introduces suggestion to find possible real examples.

**Remark 2.6.** It may be noted that we can find correct examples if [Definition 3.1, 1] satisfied for  $i \neq j$ ,  $i, j = 1$  or  $2$ .

Kandil et al. [Example 3.8, 1] asserted that the collection  $\mathcal{I} = \{\emptyset, (1, \infty), (a, \infty), [a, \infty), (a, b), [a, b), (a, b], [a, b], \{c\}\}$ , where  $1 < a < b$ ,  $1 < c < \infty$  is ideal and build their example on this assertion, but this is wrong by the following remark.

**Remark 2.7.** If  $1 < a < b < c$  or  $1 < c < a < b$ , then  $(a, b), [a, b), (a, b], [a, b], \{c\} \in \mathcal{I}$  but  $(a, b) \cup \{c\}, [a, b) \cup \{c\}, (a, b] \cup \{c\}, [a, b] \cup \{c\} \notin \mathcal{I}$ . As a consequence,  $(\mathbb{R}, \tau_l, \tau_u, R, \mathcal{I})$  is not ideal bitopological ordered space and the example is invalid.

The following remark shows that the collection  $\mathcal{I} = \{\emptyset, (0, \infty), (a, \infty), [a, \infty), (a, b), [a, b), (a, b], [a, b], \{c\}\}$ , where  $0 \leq a < b$ ,  $0 \leq c < \infty$  presented in [Examples 3.9 and 3.10, 1] is not ideal.

**Remark 2.8.** If  $0 \leq a < b < c$  or  $0 \leq c < a < b$ , then  $(a, b), [a, b), (a, b], [a, b], \{c\} \in \mathcal{I}$  but  $(a, b) \cup \{c\}, [a, b) \cup \{c\}, (a, b] \cup \{c\}, [a, b] \cup \{c\} \notin \mathcal{I}$ . As a consequence,  $(\mathbb{R}, \tau_u, \tau_l, R, \mathcal{I})$  and  $(\mathbb{R}, \tau_u, \tau_u, R, \mathcal{I})$  are not ideal bitopological ordered spaces and the examples are invalid. Moreover, the authors asserted in [Example 3.10, 1] that  $A = (1, \infty)$  and  $B = (-\infty, 0)$  are two  $\mathcal{P}$ -separated sets but this is totally wrong as  $\tau_u - cl(A) = \mathbb{R}$  and hence  $\tau_u - cl(A) \cap B = (-\infty, 0) \neq \emptyset$ .

Kandil et al. [Example 3.11, 1] claimed that the subsets  $A = \{2, 3\}$  and  $B = \{1\}$  are  $\mathcal{IP}$ -separated and construct their example on this assertion, but this is incorrect by the following remark.

**Remark 2.9.**  $\tau_2 - cl(A) = X$  and hence  $\tau_2 - cl(A) \cap B = \{1\} \notin \mathcal{I}$ . That is  $A$  and  $B$  are not  $\mathcal{IP}$ -separated sets. As a result, the example is invalid.

**Remark 2.10.** It may be noted that Example 3.11 in [1] is correct if the authors stated that [Definition 3.5, 1] satisfied for  $i \neq j$ ,  $i, j = 1$  or  $2$ .

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